

Factorising Quadratic Trinomials by Completing the Square

Example 1:

$$x^2 - 10x + 23$$

$$= (x^2 - 10x + 25) - 25 + 23$$

$$= (x^2 - 10x + 25) - 2$$

$$= (x^2 - 10x + 25) - 2$$

$$= (x - 5)^2 - 2$$

$$= (x - 5 + \sqrt{2})(x - 5 - \sqrt{2})$$

$$ax^2 + bx + c$$

Step 1: Take out any common factors.

Step 2: Complete the square:

- half b,
- square it
- Add it on
- Take it off

Step 3: Factorise the perfect square

Step 4: Factorise: DOTS

Step 5: Simplify

Example 2:

$$3x^2 + 42x + 120$$

$$= 3[x^2 + 14x + 40]$$

$$= 3[(x^2 + 14x + 49) - 49 + 40]$$

$$= 3[(x^2 + 14x + 49) - 9]$$

$$= 3[(x^2 + 14x + 49) - 9]$$

$$= 3[(x + 7)^2 - 9]$$

$$= 3[(x + 7 + 3)(x + 7 - 3)]$$

$$= 3[(x + 10)(x + 4)]$$

$$= 3(x + 10)(x + 4)$$

Factorising Quadratic Trinomials by Completing the Square: More Examples

Example 3:

$$x^2 - 5x - 36$$

$$\left(x^2 - 5x + \left(\frac{5}{2}\right)^2\right) - \left(\frac{5}{2}\right)^2 - 36$$

$$\left(x^2 - 5x + \left(\frac{5}{2}\right)^2\right) - \frac{25}{4} - 36$$

$$\left(x^2 - 5x + \left(\frac{5}{2}\right)^2\right) - \frac{25}{4} - \frac{144}{4}$$

$$\left(x^2 - 5x + \left(\frac{5}{2}\right)^2\right) - \frac{169}{4}$$

$$\left(x - \frac{5}{2}\right)^2 - \frac{169}{4}$$

$$\left(x - \frac{5}{2} - \frac{13}{2}\right)\left(x - \frac{5}{2} + \frac{13}{2}\right)$$

$$\left(x - \frac{18}{2}\right)\left(x + \frac{8}{2}\right)$$

$$(x - 9)(x + 4)$$

Example 4:

$$2x^2 - 5x - 3$$

Must take any "a" not equal to 1 out as common factor.

$$2\left[x^2 - \frac{5}{2}x - \frac{3}{2}\right]$$

$$2\left[\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) - \frac{25}{16} - \frac{3}{2}\right]$$

$$2\left[\left(x - \frac{5}{4}\right)^2 - \frac{49}{16}\right]$$

Factorise perfect square, and simplify fractions

$$2\left[\left(x - \frac{5}{4} - \frac{7}{4}\right)\left(x - \frac{5}{4} + \frac{7}{4}\right)\right]$$

DOTS

$$2\left[\left(x - \frac{12}{4}\right)\left(x + \frac{2}{4}\right)\right]$$

$$2\left[(x - 3)\left(x + \frac{1}{2}\right)\right]$$

$$(x - 3)(2x + 1)$$

Expanded the "2" back into one of the brackets just for fun.

Example 5:

$$3x^2 - 7x - 5$$

$$3\left[x^2 - \frac{7}{3}x - \frac{5}{3}\right]$$

$$3\left[\left(x^2 - \frac{7}{3}x + \left(\frac{7}{6}\right)^2\right) - \left(\frac{7}{6}\right)^2 - \frac{5}{3}\right]$$

$$3\left[\left(x^2 - \frac{7}{3}x + \frac{49}{36}\right) - \frac{49}{36} - \frac{5}{3}\right]$$

$$3\left[\left(x - \frac{7}{6}\right)^2 - \frac{109}{36}\right]$$

$$3\left(x - \frac{7}{6} - \frac{\sqrt{109}}{6}\right)\left(x - \frac{7}{6} + \frac{\sqrt{109}}{6}\right)$$

$$3\left(x - \frac{7 + \sqrt{109}}{6}\right)\left(x - \frac{7 - \sqrt{109}}{6}\right)$$